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Technical Notes

Eigensensitivity Analysis in Variable Geometry Trusses

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I. Introduction

THE adaptive structures of variable geometry are light mechanical systems capable of modifying their geometry and mechanical properties to adapt to different operating conditions. There are different kinds of mechanical variable geometry systems [1], among which stand out the variable geometry trusses (VGTs) [2–5]. These structures are a subset of the former, generally comprising a large number of biarticulated bars to form a complex truss. Some of these bars are active elements; that is, their length can vary in a controlled way to enable the actuation of the VGT.

One of the most important problems to be solved in VGTs is vibration, essentially due to two factors. On the one hand, they are generally slimline structures, making them easily excitable at low frequencies with large amplitudes, which might interfere with correct VGT operation, thereby harming its accuracy, and even resulting in collisions with environmental obstacles [5–7]. On the other hand, as VGTs may have highly different configurations throughout their operation, the dynamic properties (natural frequencies and vibration modes) also change to a large extent.

There have been numerous contributions on vibratory dynamics of VGTs. There are a few works, like those of Keane and Bright [8], Keane [9], and Nair and Keane [10], which opt for optimum redesign of the structure geometry using evolutionary methods to determine structural geometries presenting a better dynamic response; the most widely accepted procedure for improving and/or controlling dynamic properties is the inclusion of active, semiactive, or passive elements to control its dynamic response. In this sense, Bilbao et al. [11] carried out a detailed revision of the state of the art and proposed a new methodology for the optimal location of damping elements.

This Note further contributes to vibratory dynamics of VGTs, describing a tool developed to efficiently estimate the dynamic properties of VGTs (frequencies and modes) throughout their movement, by responding to two needs. First, the information that frequencies and modes supply is often sufficient to foresee the dynamic behavior without having to perform costly direct dynamic analyses, as verified in [12]. Second, should said analyses be necessary, they can be approached from the modal superposition viewpoint, for which a tool that efficiently estimates natural frequencies and vibration modes is necessary. A linear estimation is proposed to model the variation of these dynamic properties throughout the VGT evolution, so that there is no need to calculate them per position but only for a fraction of these positions, which may result in considerable computational saving. Thus, a methodology estimating the value of natural frequencies and vibration modes in a new position from their value in the immediately prior position is employed. This is done by differentiating natural frequencies and vibration modes with respect to nodal coordinates at the starting position and developing a first-order series around the starting position to extrapolate the values of natural frequencies and vibration modes for a new position.

II. Finite Element Model for Variable-Geometry-Truss Dynamic Analysis

For global dynamic analysis of a VGT, the use of simplified finite element models (FEMs) [13] is recommended, based on bar-type elements joined together via spherical joints, as can be seen in Fig. 1a. The simplicity of these models relies on the fact that localization of the FEM nodes matches with that of the joints, for which the coordinates are included in a vector $\{x\}$.

The mass matrix of the FEM is lumped instead of consistent, remaining constant in different positions, since it does not depend on nodal coordinates. The stiffness matrix can be expressed using the geometric matrix [13]

$$[K(\{x\})] = \sum_{e=1}^{b} k_e [\bar{g}_e(\{x\})] \tag{1}$$

Where b is the number of structure bar elements (actuators included), k_e is the stiffness of each bar/actuator element, and $[\bar{g}_e]$ is the elemental geometric matrix $[g_e]$ expanded to all the model degrees of freedom and global coordinates [6,7,11–13].

III. Eigenvalue and Eigenvector Estimation

A. Eigenvalue and Eigenvector Derivatives

VGTs are used to work at low velocities, accelerations, and loads. Thus, the effect of the complementary matrices (centrifugal stiffness, spin stiffness, Coriolis, stress stiffening, and indirect forces) and other effects are negligible. To obtain frequencies and natural modes in a mechanical system, the calculation equation of eigenvalues and eigenvectors [14] (squares of natural frequencies and modal vectors, respectively) is proposed:

$$([K] - \lambda_i [M])\{\varphi\}_i = \{0\} \tag{2}$$

where $[K]$ is the stiffness matrix, and $[M]$ is the matrix of masses characterizing the structure. The equation $\lambda_i = \omega_i^2$ denotes the eigenvalues corresponding to the squares of natural frequencies, and $\{\varphi\}_i$ are eigenvectors, which correspond to the natural vibration modes. Considering that the concentrated matrix of masses does not vary with nodal coordinates $\{x\}$, the derivative with respect to

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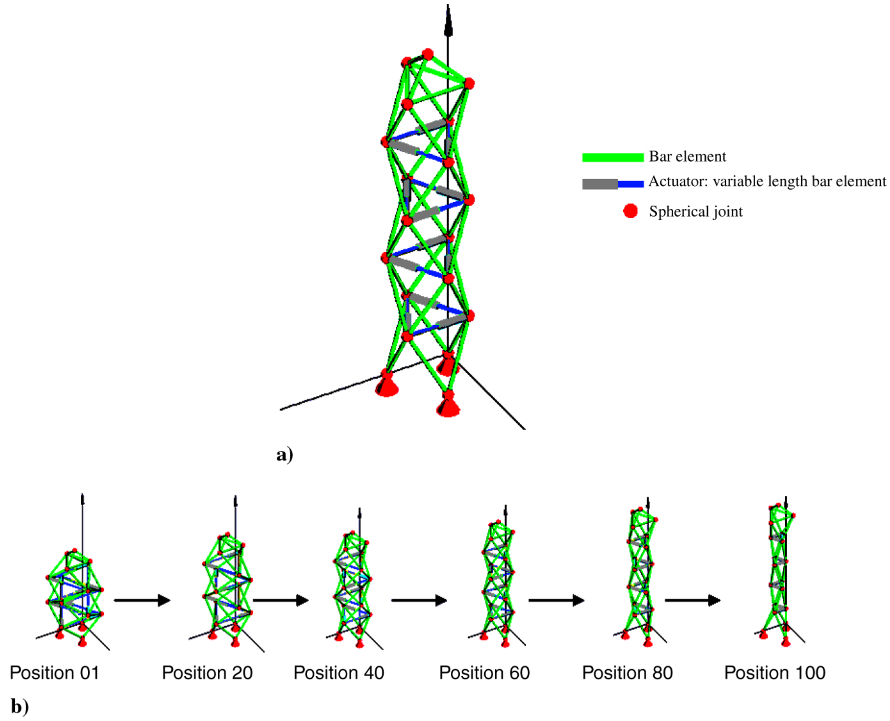


Fig. 1 VGT: a) with 12 actuators, and b) deployment operation.

nodal coordinates may be expressed vectorially, as is described in [15]:

$$\frac{\partial \lambda_i}{\partial\{x\}} = \frac{\partial \omega_i^2}{\partial\{x\}} = \left\{ \begin{array}{c} \{\varphi\}_i^T \left[\frac{\partial[K]}{\partial x_1}\right] \{\varphi\}_i \\ \vdots \\ \{\varphi\}_i^T \left[\frac{\partial[K]}{\partial x_k}\right] \{\varphi\}_i \\ \vdots \\ \{\varphi\}_i^T \left[\frac{\partial[K]}{\partial x_N}\right] \{\varphi\}_i \end{array} \right\} \tag{3}$$

Different methods to calculate the derivatives of natural modes have been proposed in previous work: for instance, the one developed by Fox and Kapoor in 1968 [15]. In that work, Fox and Kapoor present two alternative approaches: one of them uses modal superposition and the other does not. The first one is computationally economic when used with a truncated modal superposition. Regarding the second one, in 1976, Nelson [16] proposed a simplified procedure to solve it; nevertheless, although it only requires the eigenvector for which the differentiation is to be done, it is still computationally expensive. In 1991, Wang [17] proposed a static correction term to the truncated solution. Therefore, the authors have adopted here the first procedure of Fox and Kapoor [15], including Wang's [17] static correction. Particularizing the case of the concentrated mass matrix, the Wang static correction term is applied in combination with the Fox and Kapoor method [15] [Eq. (4)]:

$$\left. \begin{array}{c} \frac{\partial\{\varphi\}_i}{\partial x_k} \approx \sum_{j=1}^{p} a_{ji}\{\varphi\}_j + \delta_i\{w\}_i \\ [K]\{w\}_i = -\left(\frac{\partial[K]}{\partial x_k} - \frac{\partial\lambda_i}{\partial x_k}[M]\right)\{\varphi\}_i - ([K]-\lambda_i[M])\left(\sum_{j=1}^{p} a_{ji}\{\varphi\}_j\right) \to \{w\}_i \\ \delta_i = \frac{-\{w\}_i^T(\{\partial[K]/\partial x_k\} - (\partial\lambda_i/\partial x_k)[M])\{\varphi\}_i}{\{w\}_i^T([K]-\lambda_i[M])\{w\}_i} \end{array} \right\} \tag{4}$$

To calculate the derivatives of modes and frequencies, the stiffness matrix derivatives must be calculated with respect to the nodal coordinates $\{x\}$.

B. Linear Estimation of Frequencies and Modes

If the natural frequencies are known for a position $\{x\}$, one can obtain a linear estimate of the natural frequencies for a new position $\{x\} + \Delta\{x\}$:

$$\omega_i^2|_{\{x\}+\Delta\{x\}} = \bar{\lambda}_i|_{\{x\}+\Delta\{x\}} = \lambda_i|_{\{x\}} + \Delta\{x\}^T \left\{\frac{\partial \lambda_i}{\partial\{x\}}\right\}\bigg|_{\{x\}} \tag{5}$$

The same can be done with the vibration modes:

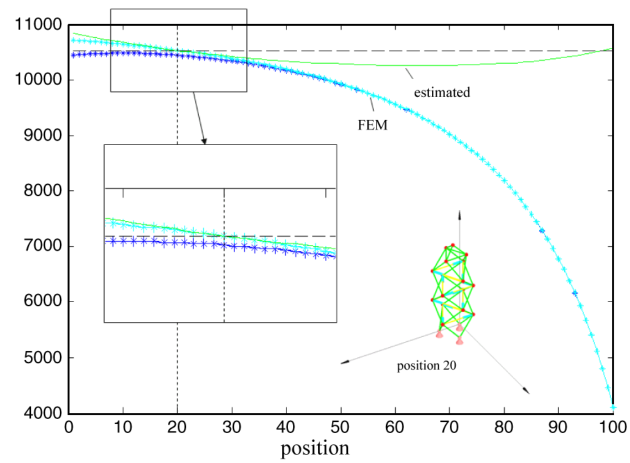
$$\{\bar{\varphi}\}_i|_{\{x\}+\Delta\{x\}} = \{\varphi\}_i|_{\{x\}} + \left[\frac{\partial\{\varphi\}_i}{\partial\{x\}}\right]^T\bigg|_{\{x\}} \Delta\{x\} \tag{6}$$

To assess the quality of the estimate, two criteria are proposed: Criterion 1:

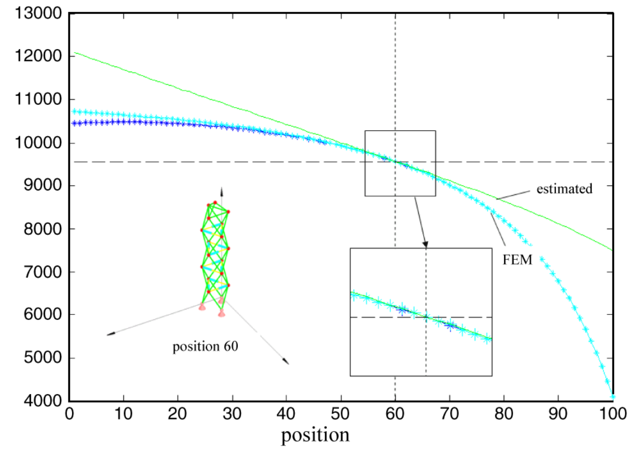
$$\sum_i (\det|[K]_{\{x\}+\Delta\{x\}} - \bar{\lambda}_i[M]|)^2 \leq \varepsilon \tag{7}$$

Criterion 2:

$$\sum_i \|([K]_{\{x\}+\Delta\{x\}} - \bar{\lambda}_i[M])\{\bar{\varphi}\}_i\|^2 \leq \varepsilon \tag{8}$$

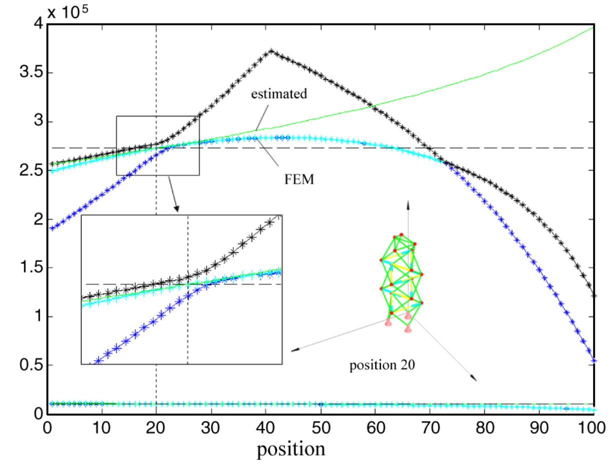


a)



b)

Fig. 2 Second natural frequency estimated from a) position 20 and b) position 60.



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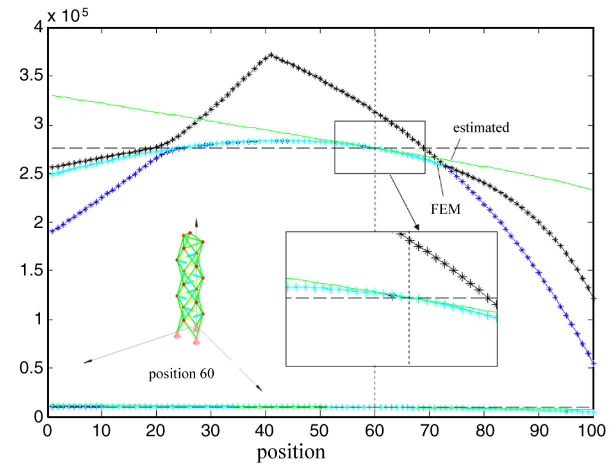

b)

Fig. 3 Fourth natural frequency estimated from a) position 20 and b) position 60.

IV. Application Example

Figure 1a shows a VGT [9] with 12 actuators. The lower natural frequencies are computed during the deployment operation illustrated in Fig. 1b. Structure bar lengths range from 0.6 to 1.5 m. All bars have a mass of 0.936 kg and a constant stiffness of 2.52×107 N/m.

A. Estimation of Natural Frequencies

A solution has been achieved for a structure deployment problem throughout 100 successive positions. Figure 2 shows the evolution of the first and second natural frequencies (FEM values) and the estimate of the second one in positions 20 and 60. Figure 3 shows the evolution of the lower five natural frequencies and includes estimates for the fourth one, also in positions 20 and 60.

Regarding the two lower natural frequencies, one can see from position 40 that they practically coincide. Nevertheless, the modes they correspond to are very different. As a conclusion, it can be stated that the adequacy of the estimations is excellent within a range of approximately 10 positions around a given value of the natural frequency. This means that throughout the 100 positions along the deployment, the natural frequencies and vibration modes need only be calculated 10 times to carry out the modal superposition. Table 1 shows the adequacy percentages of the linear estimation in relation to the natural frequency calculated with the FEM in the four cases described and for 10 positions around the calculation position. The adequacy percentage is defined as

$$\eta = \left(1 - \frac{|\lambda_{\text{real}} - \lambda_{\text{estimated}}|}{\lambda_{\text{real}}}\right) \times 100 \tag{9}$$

The values appearing in Table 1 are the mean value of the previous percentage calculated for 10 previous and 10 subsequent positions.

B. Estimation of Vibration Modes

Here, the validity of the estimation for the second eigenmode is verified for position 40 from its value in position 20. The Fox and Kapoor method [15] is used to calculate the modal vector derivatives. The accuracy of the estimations is compared for different levels of truncation, including the case of complete modal superposition. Likewise, the improvement achieved through the static Wang correction is also checked [17]. It was deemed more convenient to use the specific technique detailed next instead of resorting to a standard method of modal assurance criterion to assess estimation accuracy in this case.

To quantify the mode geometry variation from positions 20 to 40, expression (10) is used, which represents the variation percentage of the modal vector on modifying the position:

$$\Delta_{\text{ref}} = \frac{1}{2}\left\| \frac{\{\varphi\}_{2(20)}}{\|\{\varphi\}_{2(20)}\|} - \frac{\{\varphi\}_{2(40)}}{\|\{\varphi\}_{2(40)}\|} \right\| \times 100 \tag{10}$$

Table 1 Adequacy percentages α of the estimation

Adequacy percentages η	Second natural frequency, %	Fourth natural frequency, %
Position 20	99.68	99.28
Position 60	99.17	98.77

Table 2 Adequacy percentages and correction of mode estimate

Analysis	γ_1, %	γ_2, %
Fox and Kapoor[a] 5 modes	97.76	4.67
Fox and Kapoor[a] 10 modes	97.77	5.10
Fox and Kapoor[a] 15 modes	98.82	49.89
Fox and Kapoor[a] 20 modes	99.23	67.09
Fox and Kapoor[a] 57 modes (all)	99.69	86.62
Fox and Kapoor[a] 10 modes and Wang[b] correction	99.69	86.62
Fox and Kapoor[a] 5 modes and Wang[b] correction	99.69	86.62

[a]See [15].
[b]See [17].

where $\{\varphi\}_{i(j)}$ corresponds to mode i in position j. In this case, $\Delta_{\text{ref}} = 2.36\%$, which will be taken as a reference to measure estimation quality. To quantify estimation quality, a procedure analogous to that used for natural frequencies may be used, defining the adequacy percentage γ_1 as

$$\gamma_1 = (1 - \Delta) \times 100 \tag{11}$$

where

$$\Delta = \frac{1}{2}\left\| \frac{\{\bar{\varphi}\}_{2(40,20)}}{\|\{\bar{\varphi}\}_{2(40,20)}\|} - \frac{\{\varphi\}_{2(40)}}{\|\{\varphi\}_{2(40)}\|} \right\| \times 100 \tag{12}$$

Here, $\{\varphi\}_{i(j)}$ corresponds to mode i in position j, and $\{\bar{\varphi}\}_{i(k,j)}$ is mode i estimated in position k from position j. However, to decide which truncation degree is adequate, a correction percentage γ_2 must be defined besides the adequacy percentage γ_1; this correction percentage contains quantitative information about how much changes the FEM mode, and it is intended to measure the merit of the estimation:

$$\gamma_2 = \left(1 - \frac{\Delta}{\Delta_{\text{ref}}}\right) \times 100 \tag{13}$$

Table 2 shows adequacy and correction percentages for different truncation levels applying the Fox and Kapoor method [15].

The results summarized in Tables 1 and 2 show the excellent behavior of the Wang correction [17] and the approach used in this Note.

V. Conclusions

This work presents a procedure for estimating the variation of natural frequencies and vibration modes of VGTs during their movement, making it unnecessary to recalculate them at all positions but just for a small number thereof. It is possible to efficiently use modal superposition techniques with it, thus avoiding costly dynamic analysis based on direct integration techniques. Furthermore, it allows direct knowledge of the contribution of each mode in the structure movement, which is of special interest in VGT design.

To model the variations of frequencies and modes throughout VGT evolution, linear estimation has been used, which obtains natural frequencies and vibration modes in a new position from their values in a previous position. For this, the derivatives of the natural frequencies and vibration modes are calculated in relation to the nodal coordinates for a reference position. Subsequently, a first-order series is developed around this position to obtain a precise estimation in the new position.

To obtain the vibration mode derivatives, the Fox and Kapoor method [15] was adapted for use with the Wang static correction [17] in the case of VGTs. The results presented in the example prove the excellent behavior of this approach.

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